A MULTI-COMMODITY FLOW APPROACH TO THE CREW ROSTERING PROBLEM

by

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ABSTRACT

An airline crew rostering problem is a large-scaled and complex optimization problem that assigns crew members to the flight duties while satisfying agreements with the labor union, the government regulations, the carrier’s own policies, and other requirements. The traditional crew rostering problem considers only minimizing the total per-diem in order to reduce the airline expense. This paper presents the crew rostering problem for the international flights of Thai Airways with the major concern on the crew’s life quality. We propose a 0-1 multi-commodity flow problem whose objective function is to balance the per-diem and workloads among the crew members. Various test cases are generated from Thai Airways data set and solved by using the commercial optimizer, IBM ILOG CPLEX. The quality of the solutions and computational times are presented and also discussed the results.

KEYWORDS
Crew Rostering, Multi-Commodity Flow Problem

INTRODUCTION

The crew cost, which includes salaries and expenses, is the second most significant cost for an airline, behind the cost of fuel. The more an airline reduces costs, the more profitable an airline gets. Therefore efficient personnel planning such as crew scheduling plays an essential role in reducing costs and has posed a challenge to the researchers for the past decades.

The airline crew scheduling problem is usually divided into two processes. In the first process, known as 
pairing, the (anonymous) flying activities are created. The flight timetable is taken as an input to form the sequences of flights which known as pairings or trips. The main objective in this step is to use the minimum number of resources (crews) to schedule. In the second process, known as rostering, the goal is to assign pairings to named individuals so that the assignments cover all works (pairings) as well as the training requirements, vacations etc.

The airline crew scheduling problem has received much attention during the past decades. Therefore there were several solution methods and problem instances. Crew scheduling problem is not only essential in the airline industry but also in the transportation and health care industry. The airline crew scheduling problems have been traditionally formulated as set covering problems or set partitioning problems (Yan and Lin, 1997; Ryan, 1992). However, there are several other approaches such as nonlinear multi-commodity network flow (Desaulniers et al, 1997), heuristics approach (Ozdemir, Mohan, 2001; Manandhar et al, 2004) and the multicommodity flow (Cappanera and Gallo, 2004; MOZ and PATO, 2004). Kato and Jeenanunta (2010) proposed the set partitioned model for solving Thai domestic Airline with the main goal of the quality of life of the crew members.
This paper aims to formulate the crew rostering problem of Thai national airline called Thai Airways. The objective function of crew rostering of the Thai Airways problem is to balance the per-diem and workload rating among the crew members.

This paper is organized as follow. The crew rostering in Thai Airways will be described in section 1. The review of the network flow models in the literature is presented in section 2. In section 3 we describe Thai Airways crew rostering problem and present the corresponding multi-commodity network model. In section 4 we present the result and discussion in our case study. Finally, conclusions of study are shown in section 5.

CREW ROSTERING IN THAI AIRWAYS

There are two types of services in Thai Airways: International services and domestic services. The airline contains six types of aircraft, including B747-400’s, A340’s, B777’s, A330-600’s, and B737-400’s. Their routes can be grouped as: North America route, Europe route, Africa route, Australia and New Zealand route, Regional Routes and Domestic Services route. There are six categories of crew members: in-flight manager (IM), Air purser (AP), F who works only on the first class, E who works on the business class, R who works on the business class, Y who works on the economic class. First, we make a note on our definition of some important terms:

1. **A Block Time/Flight Time** is the time period from the aircraft starts moving from the parking at the departure airport until the aircraft stops moving at the arrival airport.
2. **A Flight Duty Period** is the period of the time from one hour before the block time until thirty minutes after the end of the block time.

The regulations for cabin attendants of Thai Airways are listed as follow:

1. Every 7 consecutive days, the total block time for each cabin crew should not exceed 34 hours.
2. Every 28 consecutive days, the total block time for each cabin crew should not exceed 110 hours.
3. Every 365 consecutive days, the total block time for each cabin crew should not exceed 1000 hours.
4. A cabin crew scheduled to a flight duty period of less than 8 hours must then immediately be given a scheduled rest period of at least 8 consecutive hours.
5. A cabin crew scheduled to a flight duty period of more than 8 hours but no more than 10 hours must then immediately be given a scheduled rest period of at least 10 consecutive hours.
6. A cabin crew scheduled to a flight duty period of more than 10 hours but no more than 12 hours must then immediately be given a scheduled rest period of at least 12 consecutive hours.
7. A cabin crew scheduled to a flight duty period of more than 12 hours but no more than 14 hours must then immediately be given a scheduled rest period of at least 14 consecutive hours.
8. A cabin crew scheduled to a flight duty period of more than 14 hours but no more than 16 hours must then immediately be given a scheduled rest period of at least 16 consecutive hours.
9. A cabin crew scheduled to a flight duty period of more than 16 hours but no more than 20 hours must then immediately be given a scheduled rest period of at least 24 consecutive hours.

The objective of a crew rostering problem of Thai Airways is to consider life quality of crew members and workload balancing. For these reasons, the problem becomes a multi-objective optimization.

NETWORK FLOW MODEL

During the past decades, there has been much research on crew scheduling. Traditionally, the airline crew scheduling problem has been formulated as a set covering problem or a set partitioning problem (Ryan, 1992 ; Yan and Lin, 1997). However in this section we proposed the network model and multi-commodity network flow models for solving crew scheduling.

Yan, S. et al (2002) presented models that incorporate three factors: home bases, aircraft, or cabin classes, into the crew scheduling problem in order to improve the construction of cabin crew schedules. The authors develop eight models for minimizing crew cost and planning proper pairings under the real constraints for a Taiwan airline. The networks are constructed using weekly flight schedules and cabin crew information. The eight models were formulated as integer linear programs which are solved by a column-generation-based algorithm constructed developed by the authors.
Yan S. & Tu Y.-P. (2002) constructed pure network models which can both efficiently and effectively solve crew scheduling problems for a Taiwan airline. The flow decomposition method (Ahuja et al., 1993) was used to generate pairings that cover all duties.

Cappanera and Gallo (2004) formulated the airline crew rostering problem as a 0-1 multicommodity flow problem and focused on the minimization the number of noncovered activities in the objective function. They used a preprocessing phase in order to reduce the size of the network and proposed some families of valid inequalities that had proved to be computationally effective.

Desaulniers et al (1997) has proposed two models for Daily Aircraft Routing and Scheduling. The first model is constructed as a set partitioning type formulation and a column generation method was employed to solve the linear relaxation of the set partitioning problem. The second model is constructed as a time constrained multicommodity flow formulation and a Dantzig-Wolfe decomposition method was used to solve the linear relaxation of the problem. Finally a branch-and bound algorithm was used to obtain integer solutions of the two models.

Moz and pato (2004) presented two new integer multicommodity flow formulations for solving the problem of rerostering nurse schedules. The authors do not specify the technique used for solving this problem, but focus on the comparison of the two models in terms of solution quality and computational time. The first model is based on a directed multilevel acyclic network. The second model is obtained by aggregating some nodes in the first model.

THAI AIRWAYS CREW ROSTERING WITH MULTI-COMMODITY NETWORK MODEL

The formulation of the multi-commodity network flow model for Thai Airways crew rostering problem can be formulated on a directed graph $G= (N, A)$, where $N$ is the set of nodes and $A$ is the set of arcs.

We construct a network that satisfies rules and regulations of Thai Airways. Moreover, we assume that the crew pairing problem has been solved. There are six kinds of crew members of Thai Airways that we mentioned earlier. However in our case study, we consider only the crew rostering problem for the in-flight managers.

Figure 1 shows an example of the multi-commodity network flow model that represents possible scheduling for 2 working days and 4 pairings. Every node represents a point in time. There are four types of arcs which are day-off arcs, duty arcs, rest arcs, and a cyclic arc.

1. **Day-off arcs**: a day-off arc represents a day off. The head node and the tail node of a day-off arc indicate the beginning time of the day, which is 0:00 and the ending time of the day, which is 24:00, respectively. The arc cost is zero. The lower bound is zero and the arc upper bound is infinite.

2. **Duty arcs**: a duty arc represents a work duty for a pairing. The arc cost is given and depends on the corresponding pairing. The arc lower bound and upper bound are equal to one, meaning that exactly one crew member serves as IM for that duty.

3. **Rest arcs**: the rest arcs connect the head node of a day-off arc with all starting nodes of the duty arcs of the same day. The rest arcs also connect all ending nodes of the duty arcs with tail nodes of the day-off arc of the same day. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.

4. **Cyclic arc**: a cyclic arc connects the end schedule node to the start schedule node. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.

In the crew rostering network, we build the rostering network as follows:

1. The nodes of a day-off arc are constructed in each day and the nodes will be linked by the day-off arcs. The number of day-off arcs is exactly the total number of days covered by all pairings.
2. The duty arcs are constructed according to the pairing data.
3. The rest arcs are constructed by joining the head node of a day-off arc with all starting nodes of the duty arcs of the same day. The rest arcs also connect all ending nodes of the duty arcs with tail nodes of the day-off arc of the same day.
4. The cyclic arc connects the sink node to the source node.
The formulation of the crew rostering problem is presented in this section. First we introduce the notation that is followed by the formulation.

**Notation**

The following variables are used in the model:

- \( x_{ij} \) is the binary decision variable where \( x_{ij} = 1 \) if crew member \( c \) is assigned to arc \( (i,j) \) and \( x_{ij} = 0 \) otherwise.

- \( MW \) is the variable representing the maximal total workload assigned among the crew members.

- \( MP \) is the variable representing the maximal total per-diem assigned among the crew members.

The following parameter sets are to be used:

- \( W_{ij} \) is the workload rating for arc \( (i,j) \).

- \( P_{ij} \) is the per-diem for arc \( (i,j) \).

- \( BT_{ij} \) is the block time for arc \( (i,j) \).

- \( L_{ij} \) and \( U_{ij} \) are the lower bound and the upper bound for arc \( (i,j) \).

- \( D^\circ \) is the set of arcs where the head node lies in day \( D \).

- \( C \) is the set of crew members.

- \( DAY \) is the set of valid departure days.

**Mathematical model for network flow**

The start schedule node

The end schedule node

- (1) day-off arc

- (2) duty arc

- (3) rest arc

- (4) cyclic arc
Model

The Multi-commodity flow model for Thai Airways is given by:

\[
\text{Minimize } \sum \text{MW} + \text{MP} \quad \text{subject to} \]

\[
\sum_{(i,j), \in A} x^i_j = \sum_{(j,i), \in A} x^j_i \quad \forall i \in N, \forall c \in C \quad (2)
\]

\[
\sum_{(i,j), \in A} w^i_j x^i_j \leq \text{MW} \quad \forall c \in C \quad (3)
\]

\[
\sum_{(i,j), \in A} p^i_j x^i_j \leq \text{MP} \quad \forall c \in C \quad (4)
\]

\[
\sum_{i \in B} \sum_{(i,j), \in A} BT^i_j x^i_j \leq 2040 \quad \forall D \in DA, \forall c \in C \quad (5)
\]

\[
x^i_j = \begin{cases} 1 & (i,j) \in \text{Cyclic arc}, \forall c \in C \\ 0, 1 & (i,j) \in A, \forall c \in C \end{cases} \quad (7)
\]

The objective of this model is to minimize the upper bound of workload and per-diem in order to distribute the workload and per-diem as evenly as possible while satisfying the set of constraints, which can be explained as follows.

Flow Conservation Constraints \( (2) \) state that the amount of flow into and out of each node \( i \) must equal for each crew member.

Upper bound Workload Constraints \( (3) \) ensure that the workload of each crew member does not exceed the upper bound.

Upper bound Per-diem Constraints \( (4) \) ensure that the per-diem of each crew member does not exceed the upper bound.

Block Time Constraints \( (5) \) ensure that for each crew member \( c \) cannot work more than 2040 minutes a week on 7 consecutive days.

Arc Capacity Constraints \( (6) \) demonstrate the minimum number and maximum number of crew members required for each arc.

Cyclic Arc Constraints \( (7) \) show that the flow on the cyclic arc for each crew member must be one in order to force a flow from the end schedule node to the start schedule node.

RESULTS AND DISCUSSION

To evaluate how well the model may be applied in the real situations, we performed a small case study using data from some international flights of the Thai Airways and only focused on the in-flight manager (IM) crew members. To construct the rostering network, we used the R scripting language. The multi-commodity flow problem is solved by using IBM ILOG CPLEX 12.10.

We studies six instances of the problem. The instances are varied by number of days, number of pairs and number of crew members as shown in Table 1.
TABLE 1
INSTANCES’ DETAILS AND SOLUTIONS

<table>
<thead>
<tr>
<th>Instances’ details</th>
<th>Instances</th>
<th></th>
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<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4*</td>
<td>5*</td>
<td>6**</td>
</tr>
<tr>
<td>Number of pairs</td>
<td>35</td>
<td>65</td>
<td>49</td>
<td>112</td>
<td>140</td>
<td>217</td>
</tr>
<tr>
<td>Number of days</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
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<tr>
<td>Number of crew members</td>
<td>25</td>
<td>45</td>
<td>35</td>
<td>50</td>
<td>95</td>
<td>110</td>
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<tr>
<td>Number of nodes</td>
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<td>140</td>
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<td>236</td>
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<tr>
<td>Number of arcs</td>
<td>115</td>
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<td>Number of variables</td>
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<td>9819</td>
<td>29898</td>
<td>73532</td>
<td>123423</td>
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<tr>
<td>Optimal solution</td>
<td>MaxPerdiem (\text{MP})</td>
<td>17684.37</td>
<td>20522.77</td>
<td>17684.37</td>
<td>27509.02</td>
<td>20522.77</td>
</tr>
<tr>
<td></td>
<td>MaxWorkload (\text{MW})</td>
<td>107.25</td>
<td>117.58</td>
<td>107.25</td>
<td>168.17</td>
<td>117.58</td>
</tr>
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<td></td>
<td>Computational time (sec)</td>
<td>145</td>
<td>1345</td>
<td>168</td>
<td>102</td>
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<td></td>
<td>*Relative MIP gap tolerance 10%</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>** Relative MIP gap tolerance 13%</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Our main objective is to balance workload and per-diem among the crew members. That means the workload and per-diem assigned to the crew members should be close to the average value. Table 2 displays the detailed statistical values of the per-diem and the workload that are assigned to all crew members by the optimal solution in each instance.

TABLE 2
SOLUTION QUALITY

<table>
<thead>
<tr>
<th></th>
<th>Instances</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Per-diem</td>
<td>Standard Deviation</td>
<td>756.86</td>
<td>2578.32</td>
<td>756.86</td>
<td>2446.96</td>
<td>3045.83</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>16230.40</td>
<td>16627.65</td>
<td>16230.40</td>
<td>24779.74</td>
<td>16119.60</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>17684.37</td>
<td>20522.77</td>
<td>17684.37</td>
<td>27509.02</td>
<td>20522.77</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>15493.88</td>
<td>10698.12</td>
<td>15493.88</td>
<td>16047.19</td>
<td>10698.12</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>88.64</td>
<td>89.14</td>
<td>88.64</td>
<td>135.45</td>
<td>86.59</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>107.25</td>
<td>117.58</td>
<td>107.25</td>
<td>168.17</td>
<td>117.58</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>67.67</td>
<td>63.92</td>
<td>67.67</td>
<td>74.00</td>
<td>61.33</td>
</tr>
</tbody>
</table>

FIGURE 2
SOLUTION QUALITY OF INSTANCE 1

![Distribution of Per-diem](image-url)
From Figure 2, the distribution of the per-diem and the workload from instance 1 tends to vary from the mean value more than ± 5%, which are the preferred bounds. In fact, the results of other instances also show similar characteristics. Ideally, the variation from the optimal solution should be small but our results show otherwise. This can be explained as follows:

By inspecting data, we find that the pairs in each day are the same. Moreover, for some instances such as instance 4 and instance 5 most pairs in each day are long (at least 2 days). This increases the chance of overlapping among these pairs resulting in smaller number of selections of the next pairs. Therefore, balancing the workload and the per-diem is difficult due to the limitation of choices of pairs.

In addition, the value of per-diem in each pair does not depend on the length of duty in the pair. Even if the lengths of two pairs are comparable, the per-diem can be very different. Therefore, such the choices we have make balancing the workload and the per-diem very difficult.

Moreover, the value of workload rating is smaller than 100 while the value of per-diem ranges between 2,000 and 17,000. The difference in the scale of these two quantities is quite large. Hence, from the objective function, the significance of the workload will be dominated by the per-diem. Therefore, this could cause the big variation in the workload distribution.

Even though our objective aims to minimize the upper bounds of the workload rating and the per-diem, it does not guarantee that the variation will be small. In fact the variation is quite large due to the reasons mentioned above.

CONCLUSION

In this paper, we have studied the multi-commodity flow model for solving the Thai Airways crew rostering problem that minimizes the upper bound of the per-diem and the workload of each crew member. The main objective in crew rostering is to balance per-diem and workload as much as possible. In our case study, we focus on the in-flight manager crew members. The proposed model was tested for the solution quality on six instances. The network structure of the crew rostering problem for each instance is generated using R script language and the problem is solved by using IBM ILOG CPLEX 12.10. The results show that the distribution of the workload and the per-diem is varied greatly on each instance. It is the result from the limitation of the choices of pairs available where most of them are long flight duty pairs. Moreover, the multi-objective function makes the structure of the problem more complicated.

For the future research, the network structure underlining in this model should be investigated to utilize the existing efficient multi-commodity flow algorithm to speed up the solving time. Moreover, the additional modeling techniques should be explored to improve the solution quality.

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