A STOCHASTIC TRI-LEVEL PROGRAMMING MODEL TO MINIMIZE TOTAL COST IN A SUPPLY CHAIN PLANNING WITH UNCERTAINTY DEMAND

by

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ABSTRACT

The multi-level supply chain network planning has been considered under an uncertainty demand which make the problem more difficult to solve. However, this problem can be mathematically represented by using the principle of multi-level programming. Then a stochastic tri-level programming model has been used to obtain the optimal solutions, in order to minimize the total logistic cost. This logistic cost consists of the external supplier cost, the production cost, the inventory cost and the transportation cost. Finally, the numerical example is used to illustrate the application of the method to solve the problem.

KEYWORDS
Multi-Level Programming, Stochastic, Supply Chain Planning, Uncertainty, Optimization

INTRODUCTION

The multi-level supply chain network planning has been considered under an uncertainty demand which make the problem more difficult to solve. However, this problem can be mathematically represented by using the principle of multi-level programming. Then a stochastic tri-level programming model has been used to obtain the optimal solutions, in order to minimize the total logistic cost. This logistic cost consists of the external supplier cost, the production cost, the inventory cost and the transportation cost. Finally, the numerical example is used to illustrate the application of the method to solve the problem.
**Tri-level Programming Model**

In a tri-level hierarchical decision problem, each decision entity at one level has its objective determined and its variables in part controlled by entities at other levels. The choice of values for its variables may enable it to influence the decisions made at other levels and thereby to improve its own objective function. To describe a tri-level decision problem, a basic linear tri-level programming model can be started as follows:

For $x, y, z \in \mathbb{R}^n$, $a, b, c \in \mathbb{R}^m$, $f_i(x, y, z) \in \mathbb{R}$, $i = 1, 2, 3$

\[
\begin{align*}
\min f_1(x, y, z) &= a_1 x + b_1 y + c_1 z \\
\text{s.t.} &\quad A_1 x + B_1 y + C_1 z \leq b_1 \\
\min f_2(x, y, z) &= a_2 x + b_2 y + c_2 z \\
\text{s.t.} &\quad A_2 x + B_2 y + C_2 z \leq b_2 \\
\min f_3(x, y, z) &= a_3 x + b_3 y + c_3 z \\
\text{s.t.} &\quad A_3 x + B_3 y + C_3 z \leq b_3
\end{align*}
\]

The variables $x, y, z$ are called the top-level, middle-level, and bottom-level variable respectively, and $f_1(x, y, z), f_2(x, y, z), f_3(x, y, z)$ the top-level, middle-level, and bottom-level objective functions, respectively (Guangquan Zhang et al., 2010).

**Stochastic Programming**

Stochastic programming is a mathematical programming with a stochastic element presented in the data (J. E. Beasley, Retrieved July 20, 2010). By this, it means that:

- In deterministic mathematical programming, the data (coefficients) are known numbers.
- In stochastic programming, this data is unknown, instead it may has a probability distribution present.

In most of the real life problems in mathematical programming, the parameters are considered as random variables. Most of the problems in applied mathematics may be considered as two distinct stochastic programming problem:

- probabilistic constraints
- recourse problems.

In this study, a stochastic tri-level programming model is proposed to describe and solve the supply chain planning with the uncertainty demand of customer. This model is used to determine the optimal number of external supplier, which make a decision to produce before demand is known with a stochastic programming, by minimizing the external supplier cost in the leader level model. Then the optimal number of distribution center is determined by minimizing the inventory cost and the transportation cost from distribution center to customer in the middle level model. Lastly, the optimal number of manufacturing plant is prescribed by minimizing the production cost and the transportation cost from manufacturing plant to distribution center in the bottom level model.

**STATEMENT OF THE PROBLEM**

An integrated optimization model of a supply chain planning in logistics distribution system based on several assumptions is introduced. The assumptions of this paper are as follows:

**Assumption**

1. The multi-level supply chain network consists of the manufacturing plant, distribution center, external supplier and customer with two products.
2. Customer demand is a stochastic, with a discrete probability distribution; demand, $D_i$ with probability, $P_i$, $i = 1, 2, \ldots, S$ where they have $S$ scenarios for possible future demand.
3. The distribution center is capable of keeping inventory.
4. Some resources may be controlled by capacity of manufacturing plant.
5. The customer can be supplied by the distribution center and the external supplier.
6. The distribution center can be supplied by the manufacturing plant.
7. The objective is to minimize the sum of product cost, inventory cost, external supplier cost and transportation cost of delivering products from manufacturing plant to distribution center and from distribution center to customer.

APPLICATION

We refer to a motivating example in the work of Roghanian (2007). Consider a manufacturing enterprise involving with two plants $P_1, P_2$ and one warehouse or distribution center, with two products, A and B as seen in Figure 1.

FIGURE 1
MANUFACTURING ENTERPRISE

The objective is to minimize the external supplier cost, the production cost, the inventory cost and the transportation cost. Based on the notation in Table 1, the supply chain model may be mathematically formulated into the following constrains. There are two scenarios for the possible future demand. The details are listed as in Table 2:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{1A}$</td>
<td>Production amount A in plant $P_1$ (ton)</td>
</tr>
<tr>
<td>$Y_{2A}$</td>
<td>Production amount A in plant $P_2$ (ton)</td>
</tr>
<tr>
<td>$Y_{1B}$</td>
<td>Production amount B in plant $P_1$ (ton)</td>
</tr>
<tr>
<td>$Y_{2B}$</td>
<td>Production amount B in plant $P_2$ (ton)</td>
</tr>
<tr>
<td>$T_{1A}$</td>
<td>Transportation amount A from plant $P_1$ to DC (ton)</td>
</tr>
<tr>
<td>$T_{2A}$</td>
<td>Transportation amount A from plant $P_2$ to DC (ton)</td>
</tr>
<tr>
<td>$T_{1B}$</td>
<td>Transportation amount B from plant $P_1$ to DC (ton)</td>
</tr>
<tr>
<td>$T_{2B}$</td>
<td>Transportation amount B from plant $P_2$ to DC (ton)</td>
</tr>
<tr>
<td>$X_A$</td>
<td>Inventory holding of A in DC (ton)</td>
</tr>
<tr>
<td>$X_B$</td>
<td>Inventory holding of B in DC (ton)</td>
</tr>
<tr>
<td>$Q_A$</td>
<td>Transportation amount A from DC to customer (ton)</td>
</tr>
<tr>
<td>$Q_B$</td>
<td>Transportation amount B from DC to customer (ton)</td>
</tr>
<tr>
<td>$\theta_A$</td>
<td>Demand amount A (ton)</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>Demand amount B (ton)</td>
</tr>
<tr>
<td>$\theta_{Sh}$</td>
<td>The number of units of A to buy from the external supplier in scenario $S = 1, 2$</td>
</tr>
<tr>
<td>$\theta_{3B}$</td>
<td>The number of units of B to buy from the external supplier in scenario $S = 1, 2$</td>
</tr>
</tbody>
</table>
The customer’s demands are subject to the following constraints:

\[ \begin{align*}
\theta_A + \theta_{PA} & \geq D_{PA}, \quad s = 1.2 \\
\theta_B + \theta_{PB} & \geq D_{PB}, \quad s = 1.2
\end{align*} \]

The objective is to minimize total expected cost, where total expected cost is:

\[ \begin{align*}
\min Z = 47.4\theta_A + 71.1(P_{LA} \cdot \theta_{PA} + P_{RB} \cdot \theta_{PA}) + 36.5\theta_B + 54.8(P_{LB} \cdot \theta_{PB} + P_{RB} \cdot \theta_{PB})
\end{align*} \]

Distribution centers are subject to the following constraints:

(a) Inventory levels are limited by their overall capacity

\[ X_A + X_B \leq 500 \]

(b) Inventory levels should meet demands

\[ X_A \geq \theta_A, \quad X_B \geq \theta_B \]

(c) Transportation levels from DC to customer should meet demands

\[ Q_A \geq \theta_A, \quad Q_B \geq \theta_B \]

(d) Inventory levels should not be lower than transportation levels from DC to customer

\[ X_A \geq Q_A, \quad X_B \geq Q_B \]

The objective of the distribution center is to minimize a distribution cost which may be formulated as follows:

\[ \min_{DC} Z = 15X_A + 15X_B + 31Y_A + 2Y_B + 8.53T_A + 2.5Y_B + 9.48Q_A + 7.31Q_B \]

The production part is subjected to the following constraints:

(a) Common resources are shared by both plants

\[ Y_{LA} + Y_{LB} \leq 200, \quad Y_{LA} + Y_{LB} \leq 500 \]

(b) Some resources may be controlled by individual plant condition

\[ Y_{LA} + Y_{LB} \leq 200, \quad Y_{LA} + Y_{LB} \leq 250 \]

(c) Production levels achieved by both plants should not be lower than levels required by the inventory

\[ Y_{LA} + Y_{LB} \geq X_A, \quad Y_{LA} + Y_{LB} \geq X_B \]

(d) Transportation levels from both plants to DC should not be lower than levels required by the inventory

\[ T_{LA} + T_{LB} \geq X_A, \quad T_{LA} + T_{LB} \geq X_B \]

(e) Production levels achieved by both plants should not be lower than transportation levels from both plants to DC

\[ V_{LA} + V_{LB} \geq T_{LA} + T_{LB}, \quad V_{LA} + V_{LB} \geq T_{LA} + T_{LB} \]

The objective of the production part is to minimize a production cost and transportation cost from plants to DC which can be typically formulated as follows:

\[ \min_{P} Z = 47.4\theta_A + 71.1(P_{LA} \cdot \theta_{PA} + P_{RB} \cdot \theta_{PA}) + 36.5\theta_B + 54.8(P_{LB} \cdot \theta_{PB} + P_{RB} \cdot \theta_{PB}) \]

A way to pose this problem is formulating the problem as a tri-level programming model which is as follows:

\[ \min_{P} Z = 47.4\theta_A + 71.1(P_{LA} \cdot \theta_{PA} + P_{RB} \cdot \theta_{PA}) + 36.5\theta_B + 54.8(P_{LB} \cdot \theta_{PB} + P_{RB} \cdot \theta_{PB}) \]

\[ \text{s.t.} \]

\[ \begin{align*}
\theta_A + \theta_{PA} & \geq 200, \\
\theta_A + \theta_{PA} & \geq 150, \\
\theta_B + \theta_{PB} & \geq 200, \\
\theta_B + \theta_{PB} & \geq 230
\end{align*} \]

\[ \min_{DC} Z = 15X_A + 15X_B + 31Y_A + 2Y_B + 8.53T_A + 2.5Y_B + 9.48Q_A + 7.31Q_B \]

\[ \text{s.t.} \]

\[ \begin{align*}
X_A & \geq \theta_A, \quad X_B \geq \theta_B, \\
Q_A & \geq Q_A, \quad Q_B \geq Q_B
\end{align*} \]

\[ \begin{array}{c|c|c|c|c}
\text{Product} & \text{Scenario 1} & \text{Scenario 2} \\
\hline
A & D_{LA} = 200 \text{ with } \theta_{LA} = 0.6 & D_{LA} = 150 \text{ with } \theta_{LA} = 0.4 \\
B & D_{LB} = 200 \text{ with } \theta_{LB} = 0.7 & D_{LB} = 250 \text{ with } \theta_{LB} = 0.3 \\
\end{array} \]
SOLUTION METHOD

In this section a solution method will be presented. The tri-level Kth-best algorithm is applied to acquire a solution for this problem obtained in the series of steps (Guangquan Zhang et al., 2010):

Step 1: Solve leader problem with the simplex method, to decide how much to produce now before demand is known, get \( \bar{\theta} = (\theta_{1A}, \theta_{2A}, \theta_{1B}, \theta_{2B}) \).

Step 2: Use \( \bar{\theta} \) as an additional constraint, solve the middle level follower’s linear program with bounded simplex method, get \( X = (X_{1A}, X_{1B}) \).

Step 3: Use \( \bar{\theta}, X \) as an additional constraint, solve the bottom level follower’s linear program with bounded simplex method, get \( Y = (Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B}) \).

Step 4: Get the optimal solution for the tri-level programming to minimize the total cost.

According to the procedure and by using Lingo-12, the solutions of this problem are represented in Table 3.

<table>
<thead>
<tr>
<th>Model</th>
<th>Tri-level Programming</th>
<th>Linear Programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Value</td>
<td>19,608.5</td>
<td>24588.60</td>
</tr>
<tr>
<td>External Supplier Cost</td>
<td>5,383</td>
<td>24588.60</td>
</tr>
<tr>
<td>( \theta_{1A} )</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>( \theta_{2A} )</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>( \theta_{1B} )</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>( \theta_{2B} )</td>
<td>50</td>
<td>250</td>
</tr>
<tr>
<td>Production Cost</td>
<td>3,025</td>
<td>0</td>
</tr>
<tr>
<td>( Y_{1A} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Y_{2A} )</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>( Y_{1B} )</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>( Y_{2B} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inventory Cost</td>
<td>5,475</td>
<td>0</td>
</tr>
<tr>
<td>( X_{1A} )</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>( X_{1B} )</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>Transportation Cost</td>
<td>5,725.5</td>
<td>0</td>
</tr>
<tr>
<td>( T_{1A} )</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>( T_{2A} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( T_{1B} )</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>( T_{2B} )</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>( T_{2B} )</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>( T_{2B} )</td>
<td>200</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3 displays that the obtained solutions are different with the solution of tri-level programming is better than the solution of linear programming. So we can note that the objective function value is 19,608.5 and we assume it is a compromise solution.

CONCLUSION

In this paper, the stochastic tri-level programming model is developed to solve the supply chain planning problem with the uncertainty demand of customer by minimize the total logistic costs. The tri-level Kth-best algorithm is applied to solve the proposed model and implemented in Lingo12. Its application in decentralized supply chain planning is discussed by an example in section 3. Finally, the further research is required to study the stochastic tri-level programming with all random coefficients.

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REFERENCES


